

Maximizing a psychological uplift in love dynamics

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Abstract In this paper, we investigate the dynamical properties of a psychological uplift in lovers. We first evaluate extensively the dynamical equations which were recently given by Rinaldi *et. al.* (2013) [1]. Then, the dependences of the equations on several parameters are numerically examined. From the view point of lasting partnership for lovers, especially, for married couples, one should optimize the parameters appearing in the dynamical equations to maintain the love for their respective partners. To achieve this optimization, we propose a new idea where the parameters are stochastic variables and the parameters in the next time step are given as expectations over a Boltzmann-Gibbs distribution at a finite temperature. This idea is very general and might be applicable to other models dealing with human relationships.

1 Introduction

“Love is composed of a single soul inhabiting two bodies.” – Aristotle

“Love never dies a natural death. It dies because we don’t know how to replenish its source. It dies of blindness and errors and betrayals. It dies of illness and wounds; it dies of weariness, of witherings, of tarnishings.” – Anaïs Nin

Love – mysterious and unexplained – often forms the basis of a relationship between two persons; undoubtedly, a partnership between lovers is a time-dependent

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phenomenon. Even if a man and a woman were in deep love at some initial stages, the psychological uplift for one or both of them could eventually decay to very low-levels, and could even result in a break-up or divorce, in the worst scenario.

A simple mathematical model for the dynamics of love between a man and woman was introduced by Strogatz [2, 3] – the first attempt to model the love dynamics with the help of coupled ordinary differential equations. The idea of Strogatz was then extended by other researchers [4, 5, 6] to understand the influence of the factors like appeal, secure relation between the couple, separation for a finite time period, which are important factors to maintain the relationship. Similar type of mathematical models have been proposed and analyzed up to certain extent for triangular love by Sprott [7, 8] but the uncertainty for the final outcome remains unclear. Recently, Rinaldi *et. al.* [1] again proposed a simple dynamical model for lovers emotion to investigate a law of big hit film from the dynamical behavior of feeling in the partner for lovers. Their approach, based on a coupled differential equations, was applied to the movie ‘Gone With The Wind’ (GWTW); they found that the resulting time series of lovers’ feelings can mimic the story of the film to some extent. The differential equations contain several parameters and Rinaldi *et. al.* chose them to mimic the lives of Scarlet and Rhett, with full of ups and downs. In the romantic film GWTW, the drastic ups and downs in the lovers’ emotions indeed constituted a notable factor to attract the attention of audience and the sequences of such psychological climaxes in the film might have been a key issue in making the film a big hit, as suggested by Rinaldi *et. al.* (2013) [1].

In reality, for a married couple, such extreme ups and downs could however prove to be deterrent to the continuation a peaceful married life. Hence, from the view point of lasting partnership for lovers, especially for a married couple, one should optimize the parameters appearing in the dynamical equations to maintain the love for their partner. In other words, it would be interesting to obtain the optimum levels of the parameters in order to maintain the minimum level of love and happiness required to maintain a happy and prolonged marital life.

To this aim, we propose a simple new idea in this paper. We assume that the parameters involved with the love dynamics are not constant over the entire time period, rather they are stochastic variables and the parameters in the next time step are given as expectations over a Boltzmann-Gibbs distribution at a finite temperature. By decreasing the temperature during the dynamics of coupled equations, one can accelerate the rate of increase of the sum of feelings (and decrease the difference of feelings) of lovers at each time step. The idea is quite general and might be applicable to other models dealing with human relationships.

2 Differential equations of gross and gap for lovers’ feelings

In the original model by Rinaldi *et. al.* [1], the governing equations with respect to the feelings of lovers, denoted as x_1, x_2 , are given by two coupled non-linear ordinary differential equations:

$$\frac{dx_1}{dt} = -\alpha_1 x_1 + \rho_1 A_2 + k_1 x_2 e^{-\eta_1 x_2}, \quad (1)$$

$$\frac{dx_2}{dt} = -\alpha_2 x_2 + \rho_2 A_1 + k_2 x_1 e^{-\eta_2 x_1}, \quad (2)$$

subjected to the positive initial conditions, where the parameter α_i is *forgetting coefficient*, k_i and η_i are the parameters characterizing the measure of insecurity feelings, A_j is the measure of appeal towards x_i produced by x_j and ρ_i is a multiplicative factor representing the amount of recognition of the appeal A_j (see [1] for detailed interpretation). All the parameters involved with the model are positive. Interestingly, once we choose the initial values of x_1, x_2 , these variables remain positive.

As one can see above, that there are many parameters to be calibrated. From the engineering point of view, one could determine them by means of ‘optimization’ of some appropriate cost functions. In the following, we consider several such cost functions.

First, we introduce the following new variables, namely, the ‘gross’ S (sum) and ‘gap’ D (difference):

$$S \equiv x_1 + x_2, \quad D \equiv (x_1 - x_2)^2 = x_1^2 + x_2^2 - 2x_1 x_2. \quad (3)$$

This allows us to write:

$$x_1 = \frac{1}{2}(S + \sqrt{D}), \quad x_2 = \frac{1}{2}(S - \sqrt{D}), \quad (4)$$

where we should bear in mind that we have to consider the case $x_1 \geq x_2$ in order to have the well-defined expressions for x_1 and x_2 in terms of S and D . Of course, this condition may not always be satisfied. However, as we are focusing here on the gap D , the above choice might be indeed justified. It should be noted that the gross feelings S could be regarded as a cost function to be maximized. This is because the total degree of ‘passion’ amongst the lovers might be one of the most important quantities to make the relationship strong and durable. On the other hand, the gap the two partners’ love x_1, x_2 might determine the ‘stability’ of the relationship – namely, even if the S is high, the mutual relation could be unstable when $x_1 \gg x_2$ or $x_1 \ll x_2$. In other words, it is very hard for the lovers to continue their good relationship if only one of them expresses too much love to his/her love partner and the other partner becomes indifferent about their relationship which was established due to their love affairs. Two hypothetical cases can be considered for illustrating this.

- For young lovers, the variable S takes high values temporally; however, one person (girl or boy) suddenly loses interest and becomes indifferent. As a result, the variable D increases rapidly and the love affair (marriage) breaks down prematurely.
- For senior lovers, the variable S normally does not take a high value; however, they know each other quite well, and as a result, the feelings x_1 and x_2 are quite

similar. Hence, variable D increases and the love affair (marriage) becomes stable.

We do not have any real survey data to validate these idealized examples. Nevertheless, we consider an utility function S , which is to be maximized, and the energy function D , which is to be minimized, in order to determine the parameters appearing in the original model [1].

Then, the original equations are rewritten in terms of S and D . The equation for S is easy to obtain, and we have

$$\begin{aligned} \frac{dS}{dt} = & -\alpha_1 \frac{S + \sqrt{D}}{2} + \rho_1 A_2 + k_1 \frac{S - \sqrt{D}}{2} e^{-\eta_1(S - \sqrt{D})/2} - \alpha_2 \frac{S - \sqrt{D}}{2} \\ & + \rho_2 A_1 + k_2 \frac{S + \sqrt{D}}{2} e^{-\eta_2(S + \sqrt{D})/2} \equiv f(\boldsymbol{\theta} : S, D) \end{aligned} \quad (5)$$

$$\frac{dD}{dt} = 2(x_1 - x_2) \left(\frac{dx_1}{dt} - \frac{dx_2}{dt} \right) \equiv g(\boldsymbol{\theta} : S, D), \quad (6)$$

where $\boldsymbol{\theta} \equiv (\alpha_1, \alpha_2, \rho_1, \rho_2, A_2, A_1, k_1, k_2, \eta_1, \eta_2)$.

In the following parts, we discuss in details, the behavior of the non-linear dynamics of equations (5)-(6), within the framework of Rinaldi *et. al.* [1] model, and consider the possible optimization of the parameters. Here, we have chosen the model by Rinaldi *et. al.* just as a basic example, and in principle one could easily extend the study by taking into account much more complicated and appropriate lovers' interactions.

2.1 Some specific choices of parameters

We first examine the behavior of the differential equations (5) and (6) with respect to S and D for the case of a specific choice of parameters $\boldsymbol{\theta}$. Apparently, $D_t = 0$ is always a solution of the equation (6). In Fig. 1, we plot the S_t and D_t for two distinct initial conditions. In the left panel, we choose the initial condition so that $x_1(0) = x_2(0) = 1$, this reads $D_0 = 0$. From this panel, we easily find that the gap D is time-independently zero. On the other hand, in the right panel, we choose as $x_1(0) \neq x_2(0)$, namely, $D_0 \neq 0$. For this case, the gap D evolves in time and converges to some finite value. In Fig. 2, we show the flows (trajectories) S - D for $D_0 \neq 0$. All flows converge to $(7.61, 28.19)$.

2.1.1 Symmetric case

For symmetric case $D_t = 0$ ($x_1 = x_2$), the differential equation with respect to S is simply obtained by

$$\frac{dS}{dt} = -\alpha S + 2\rho A + k S e^{-\eta S/2}. \quad (7)$$

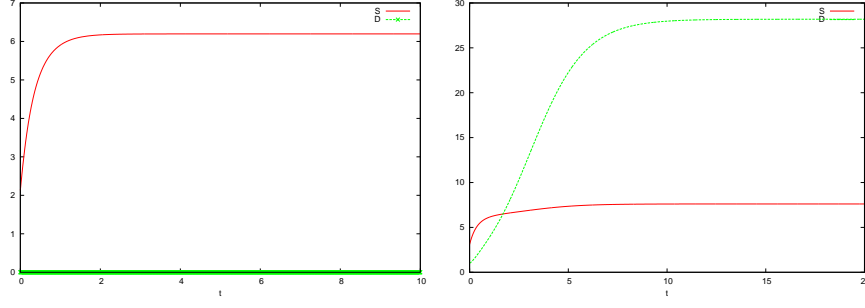


Fig. 1 The time-dependence of S and D calculated by Eqs. (5)-(6). We set the parameters as $\alpha_1 = \alpha_2 = \rho_1 = \rho_2 = \eta_1 = \eta_2 = 1$ and $k_1 = k_2 = 15, A_1 = A_2 = 1$. The initial condition are selected as $(x_1, x_2) = (1, 1)$ (left), this reads $D_0 = 0$, and $(x_1, x_2) = (2, 1)$ (right), this reads $D_0 \neq 0$.

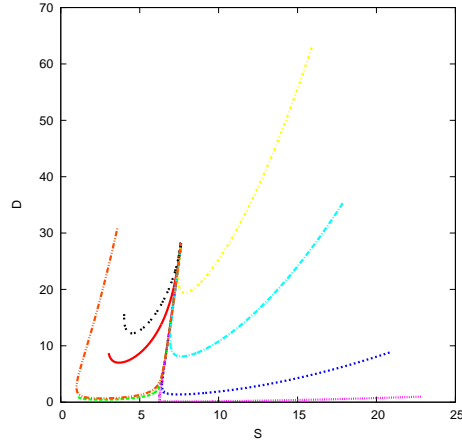


Fig. 2 The flows S - D calculated by Eqs. (5)-(6) for several distinct initial conditions. We find that for any initial conditions, the flows converge to $(7.61, 28.19)$. We set the parameters as $\alpha_1 = \alpha_2 = \rho_1 = \rho_2 = \eta_1 = \eta_2 = 1$ and $k_1 = k_2 = 15, A_1 = A_2 = 1$.

The steady state is given by the following non-linear equation.

$$\alpha S = 2\rho A + kSe^{-\eta S/2} \quad (8)$$

In Fig. 3, we show the solution S of the steady state which satisfies equation (8) as function of $1/\eta$ (left) and k (right). From this figure, we find that the S in the steady state increases monotonically in $1/\eta$ and k . As we shall discuss in the next section, from the view point of maximization of the gross S , we should increase k and $1/\eta$ to infinity. Hence, one cannot choose these parameters as finite values as $k, 1/\eta < \infty$ in the limit of $t \rightarrow \infty$.

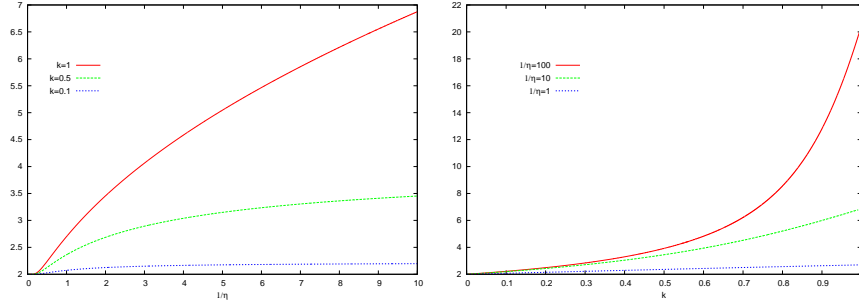


Fig. 3 The solution of the steady state which satisfies $\alpha S = 2\rho A + kSe^{-\eta S/2}$ as function of $1/\eta$ (left) and k (right). For simplicity, we set $\rho = A = 1$.

2.1.2 Breaking of symmetric phase by noise

As we saw before, as long as we choose the parameters to satisfy $A_1 = A_2, \alpha_1 = \alpha_2, \dots$, we have a symmetric solution $D_t = 0$. To break this symmetric phase, here we consider two types of additive noise, namely:

(a) Additive noise on A_1 :

$$A_1 = A_2 + \delta n, \quad n \in [-1, 1] \text{ (uniform random number)}, \quad A_2 = 1$$

(b) Additive noise on $1/\eta_1$

$$1/\eta_1 = 1/\eta_2 + \delta|n|, \quad n \in [-1, 1] \text{ (uniform random number)}, \quad 1/\eta_2 = 1$$

and change the ‘amplitude’, δ . The results are shown in Fig. 4. In this figure, we set the parameters as $\alpha_2 = \rho_1 = \rho_2 = 1/\eta_2 = 1$ and $k_1 = k_2 = 15, A_1 = A_2 = 1$. Then, we break the symmetry as $A_1 = A_2 + \delta n$ (left) and $1/\eta_1 = 1/\eta_2 + \delta|n|$ (right). The initial condition are selected symmetrically as $(x_1, x_2) = (1, 1)$.

From the left panel, we find that the symmetric phase specified by $D_t = 0$ remains up to t_c even if we add a noise on A_1 . The t_c decreases as the amplitude δ increases. On the other hand, from the right panel, we find that the symmetric phase is easily broken when we add a small noise on $1/\eta_1$. In fact, even for $\delta = 0.1$, the critical time t_c is close to zero. Moreover, we find that D_t rapidly increases when δ increases and it takes a maximum at time t_p .

3 Optimization of parameters

In the previous section, we examined the differential equations with respect to the gross S and the gap D of lovers’ feelings for some specific choice of parameters θ .

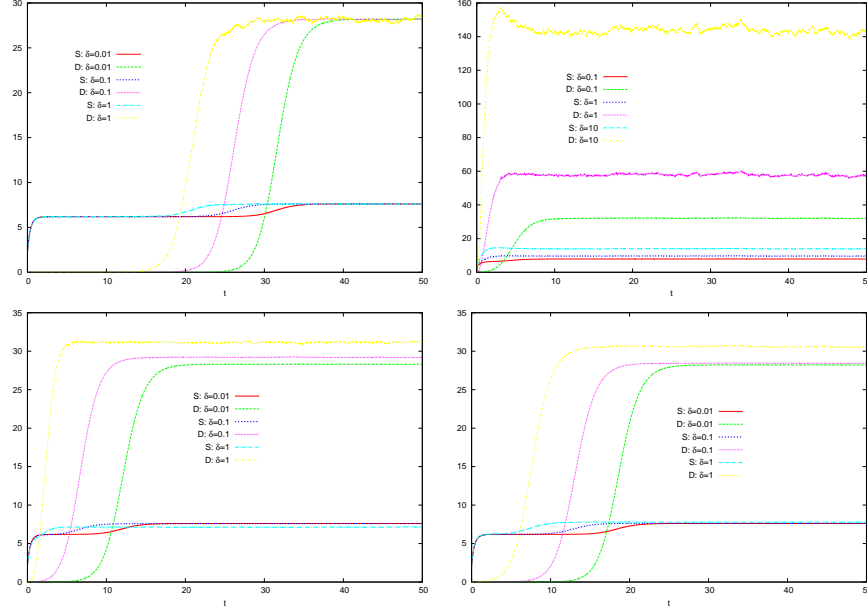


Fig. 4 The time-dependence of S and D calculated by Eqs. (5)-(6). We set the parameters as $\alpha_2 = \rho_1 = \rho_2 = 1/\eta_2 = 1$ and $k_1 = k_2 = 15, A_1 = A_2 = 1$. Then, we break the symmetry as $A_1 = A_2 + \delta n$ (left) and $1/\eta_1 = 1/\eta_2 + \delta|n|$ (right). The initial condition are selected symmetrically as $(x_1, x_2) = (1, 1)$.

In Rinaldi *et al.* [1], they chose those parameters to reproduce Scarlett O'Hara and Rhett Butler's feelings according to the fascinating plot of the movie GWTW. For this purpose, the parameters should be time-dependent because the fluctuating (up-down) behavior of main characters (Scarlett and Rhett) should be induced frequently. There is no doubt about this procedure of determining the parameters because the GWTW was historically a remarkable hit movie, and the scenario (plot) was the most important factor to make the movie success.

On the other hand, as explained earlier, in realistic situations of lovers or married couples, the persons do not have to make their feelings fluctuate (up-down) to disrupt their peaceful life and should instead enhance their psychological uplift so as to keep their affection for the partner strong. In this sense, we might treat the problem of psychology uplift for the lovers mathematically, by regarding it as an optimization or an optimal scheduling of parameters in the differential equations (5) and (6), so as to maximize the gross S and minimize the gap D , as quickly as possible.

From this viewpoint, we should solve the optimization problem for each time step t because the optimal parameters are dependent on the time step through the gross and the gap. Hence, we might choose the variables θ , so as to satisfy:

$$\theta = \arg \max_{\theta} \phi(\theta, \xi : S, D) \quad (9)$$

for each time step, where we defined the following utility function:

$$\phi(\boldsymbol{\theta}, \xi : S, D) \equiv f(\boldsymbol{\theta} : S, D) - \xi g(\boldsymbol{\theta} : S, D) \quad (10)$$

for $\xi \geq 0$. The maximization given by equation (9) means that we accelerate the speed of increase $dS/dt (= f)$ and $-dD/dt (= -g)$ as much as possible during the dynamics of S and D . Hence, when we choose the variable S for the case of $\xi = 0$ to be maximized for lovers, we should optimize the parameters appearing in the function f . For each time step, the landscape of f changes due to the dynamics of D and S , and one should choose the solution, say, $\boldsymbol{\theta} = (\alpha_1, \alpha_2, \dots)$ so as to maximize the function f at each time step. As the result, we obtain the trajectory in the parameter space: $(\alpha_1(0), \alpha_2(0), \dots) \rightarrow (\alpha_1(1), \alpha_2(1), \dots) \rightarrow \dots$.

3.1 ‘Hard’ and ‘soft’ optimizations by using a concept of physics

In the following, for simplicity, we only consider the case of $\xi = 0$ and we also carry out the maximization of the speed of increase dS/dt (see equation (5) and do not take into account the maximization of $-dD/dt$ (see equation (6)).

To achieve the parameter choice by means of physics, we start our argument from the following energy function:

$$E(\boldsymbol{\theta} : S_t, D_t) \equiv -f(\boldsymbol{\theta} : S_t, D_t). \quad (11)$$

Obviously, in terms of f , we should maximize f as a utility function. We should keep in mind that we use the definition of S_t, D_t instead of S, D to recall us that f is time dependent through those variables. We should bear in mind that the function f is defined at each time step t . In this sense, f is just a function of only parameters $\alpha_{1,2}$, etc. to be selected at each time step. Therefore, the function f is definitely conserved at each time t .

From the viewpoint of ‘hard optimization’, we might utilize the following gradient descent learning for the parameters $\boldsymbol{\theta}$ as

$$\frac{d\boldsymbol{\theta}}{dt} = -\frac{\partial E}{\partial \boldsymbol{\theta}} = +\frac{\partial f}{\partial \boldsymbol{\theta}} \quad (12)$$

Obviously, the cost function for each time step is dependent on the state (S_t, D_t) . As we mentioned in the previous section (see Fig. 4), S_t, D_t might contain some noise and through the fluctuation in S_t, D_t , the parameters $\boldsymbol{\theta}$ fluctuate around the peak of the locally concave function f . To determine the parameters, we temporarily assume here that the parameters are all ‘stochastic’ variables. Namely, to adapt ourselves to such realistic cases, we consider ensemble of the parameters and we carry out the following maximization of Shannon’s entropy under the usual two constraints of energy conservation and probability conservation:

$$H = - \int d\boldsymbol{\theta} P(\boldsymbol{\theta}) \log P(\boldsymbol{\theta}) + \beta \left(E - \int d\boldsymbol{\theta} E(\boldsymbol{\theta} : S_t, D_t) P(\boldsymbol{\theta}) \right) + \lambda \left(1 - \int d\boldsymbol{\theta} P(\boldsymbol{\theta}) \right) \quad (13)$$

where β, λ are Lagrange multipliers. By making use of derivative with respect to $P(\boldsymbol{\theta}), \lambda$, we have

$$P(\boldsymbol{\theta}) = \frac{\exp(\beta f(\boldsymbol{\theta} : S_t, D_t))}{\int d\boldsymbol{\theta} \exp(\beta f(\boldsymbol{\theta} : S_t, D_t))} \quad (14)$$

This is nothing but the Boltzmann-Gibbs distribution with temperature $T = \beta^{-1}$.

To obtain the appropriate parameters, we construct the following iterations:

$$\boldsymbol{\theta}^{(t+1)} = \int d\boldsymbol{\theta} \boldsymbol{\theta} P(\boldsymbol{\theta}) = \frac{\int d\boldsymbol{\theta} \boldsymbol{\theta} \exp(\beta f(\boldsymbol{\theta} : S_t, D_t))}{\int d\boldsymbol{\theta} \exp(\beta f(\boldsymbol{\theta} : S_t, D_t))} \quad (15)$$

We should keep in mind that the strict maximization of f is achieved by taking the limit of $\beta \rightarrow \infty$. Namely, the solution for ‘hard optimization’ is recovered as

$$\boldsymbol{\theta}_{\text{hard}}^{(t+1)} = \lim_{\beta \rightarrow \infty} \frac{\int d\boldsymbol{\theta} \boldsymbol{\theta} \exp(\beta f(\boldsymbol{\theta} : S_t, D_t))}{\int d\boldsymbol{\theta} \exp(\beta f(\boldsymbol{\theta} : S_t, D_t))}. \quad (16)$$

These types of adaptive learning procedure have been well known since the reference [9] in the literature of neural networks.

It is important for us to obtain the strict solution, of course. Hoowever, here we consider only the case of $\beta = 1$, since we are dealing with the situation in which the parameters $\boldsymbol{\theta}$ are not deterministic variables; rather, stochastic variables fluctuating around the peaks of f .

From the view point of optimization, note that the function f is not locally ‘concave’ for any choice of S_t, D_t . Hence, the parameters $\boldsymbol{\theta}$ which should be selected are trivially going to their ‘bounds’. Nevertheless in the following, we derive the concrete update rule for each parameter. We first consider the parameter α_1 . Here, we assume that $\alpha_{1,2}$ take any value in $[0, \infty)$. Hence, we obtain

$$\alpha_1^{(t+1)} = \frac{\int_0^\infty d\alpha_1 \alpha_1 e^{-\alpha_1(S_t + \sqrt{D_t})/2}}{\int_0^\infty d\alpha_1 e^{-\alpha_1(S_t + \sqrt{D_t})/2}} = \frac{2}{S_t + \sqrt{D_t}}, \quad \alpha_2^{(t+1)} = \frac{2}{S_t - \sqrt{D_t}}. \quad (17)$$

Hence, we find that the parameters $\alpha_{1,2}$ decrease as inverse of the dynamics S_t to zero, when we consider the symmetric case $D_t = 0$. Therefore, as we expected, $\alpha_{1,2}$ go to the bound $\alpha_{1,2} = 0$ but the optimal scheduling, namely, the speed of convergence to the bound $\alpha_{1,2} \sim 1/S_t$ is not trivial and would be worthwhile for us to investigate extensively.

We next consider $\rho_{1,2}$. For simplicity, we assume that these two parameters take values in $[0, 1]$. After simple algebra, we have

$$\rho_1^{(t+1)} = \frac{\int_0^1 d\rho_1 \rho_1 e^{A_2 \rho_1}}{\int_0^1 d\rho_1 e^{A_2 \rho_1}} = \frac{A_2^{(t)} e^{A_2^{(t)}} - e^{A_2^{(t)}} + 1}{A_2^{(t)} (e^{A_2^{(t)}} - 1)}, \quad \rho_2^{(t+1)} = \frac{A_1^{(t)} e^{A_1^{(t)}} - e^{A_1^{(t)}} + 1}{A_1^{(t)} (e^{A_1^{(t)}} - 1)}. \quad (18)$$

Since ρ and A are ‘conjugates’ in the argument of the exponential, we immediately have

$$A_1^{(t+1)} = \frac{\rho_2^{(t)} e^{\rho_2^{(t)}} - e^{\rho_2^{(t)}} + 1}{\rho_2^{(t)} (e^{\rho_2^{(t)}} - 1)}, \quad A_2^{(t+1)} = \frac{\rho_1^{(t)} e^{\rho_1^{(t)}} - e^{\rho_1^{(t)}} + 1}{\rho_1^{(t)} (e^{\rho_1^{(t)}} - 1)}. \quad (19)$$

For $k_{1,2}$, the structures are exactly similar to those of A and ρ , when we assume that $k_{1,2} \in [0, 1]$. We easily obtain

$$k_1^{(t+1)} = \frac{Q_1^{(t)} e^{Q_1^{(t)}} - e^{Q_1^{(t)}} + 1}{Q_1^{(t)} (e^{Q_1^{(t)}} - 1)}, \quad Q_1^{(t)} \equiv \frac{(S_t - \sqrt{D_t})}{2} e^{-\eta_1^{(t)} (S_t - \sqrt{D_t})/2}, \quad (20)$$

$$k_2^{(t+1)} = \frac{Q_2^{(t)} e^{Q_2^{(t)}} - e^{Q_2^{(t)}} + 1}{Q_2^{(t)} (e^{Q_2^{(t)}} - 1)}, \quad Q_2^{(t)} \equiv \frac{(S_t + \sqrt{D_t})}{2} e^{-\eta_2^{(t)} (S_t + \sqrt{D_t})/2}. \quad (21)$$

Finally, we consider $\eta_{1,2}$. Here we also assume that $\eta_{1,2} \in [0, \infty)$. Then, we can write

$$\eta_1^{(t+1)} = \frac{\int_0^\infty d\eta_1 \eta_1 \exp[\frac{k^{(t)} (S_t - \sqrt{D_t})}{2} e^{-\eta_1 (S_t - \sqrt{D_t})/2}]}{\int_0^\infty d\eta_1 \exp[\frac{k^{(t)} (S_t - \sqrt{D_t})}{2} e^{-\eta_1 (S_t - \sqrt{D_t})/2}]} \quad (22)$$

$$\eta_2^{(t+1)} = \frac{\int_0^\infty d\eta_2 \eta_2 \exp[\frac{k^{(t)} (S_t + \sqrt{D_t})}{2} e^{-\eta_2 (S_t + \sqrt{D_t})/2}]}{\int_0^\infty d\eta_2 \exp[\frac{k^{(t)} (S_t + \sqrt{D_t})}{2} e^{-\eta_2 (S_t + \sqrt{D_t})/2}]} \quad (23)$$

Carrying out the above procedure, one could only ‘soft’ (not ‘hard’) optimize the quantity S . By substituting the results into the differential equation with respect to D (see equation (6)) at the same time, we may obtain the behavior of the gap.

4 Discussions and remarks

In this paper, we first introduced the Rinaldi model and the framework to discuss some kind of optimality of a person’s behavior, in terms of optimization in the mathematical sense. For this, we have just formulated the acceleration rate of the gross dS/dt , namely, the right hand side of equation (5), the function f , at each time step. However, the function f is not locally concave and the value of optimal parameters go either to zero or to infinity, as $t \rightarrow \infty$. Nevertheless, we can still discuss the scheduling of parameters. For instance, the parameters $\alpha_{1,2}$ should decay as $\sim S_t^{-1}$ when we attempt to maximize the f from the viewpoint of ‘soft optimization’. In near future, we would like to consider and discuss the result of optimization extensively, by considering the validity of the model itself.

Here, we have set $\beta = 1$ in the calculations. However, we can always regard β as a time dependent parameter— the ‘inverse-temperature’, appearing in the context of ‘simulated annealing’, and defined by

$$T_t = \beta_t^{-1} = \frac{c_1}{(t + c_2)^\zeta (\log(t + c_3))^\varepsilon} \quad (24)$$

where the coefficients $c_{1,2,3}, \zeta, \varepsilon$ determine the speed of convergence. As we already mentioned, the utility function f changes through the dynamical variables S_t, D_t . Hence, the utility surface also evolves in time. In the above scheduling, we have also assumed that the temperate is decreasing within the same time scale as dynamical variables S_t, D_t and parameters θ . However, we can also consider the case in which T is scheduled in much shorter time scale than S_t, D_t and in the same time scale as θ , namely, T_τ, θ_τ with $\tau \ll t$. Then, the procedure defined by equation (16) is regarded as the “deterministic annealing” [10]. In such a general case, the optimal scheduling for the parameters θ might be changed and extensive study along this direction will be reported in our forthcoming paper.

A few other specific remarks are mentioned below:

1. In the model considered here, the parameters values are the same as those of Rinaldi *et. al.* [1], but this choice is neither unique nor true for all the “realistic” situations. A thorough study with other choices of parameters is very much necessary.
2. Identification of the most sensitive parameters responsible for the long time survival of the relationship remains an interesting and open problem. Such identification and then introduction of stochastic fluctuations at the limiting situations could certainly provide more insight towards the modelling approach.
3. The present work is sort of a preliminary attempt of understanding the love dynamics – theory for the case of sustainability of the love relation between a couple. In reality, the dynamics of love affairs and related modelling approach need more careful and thorough investigations; the effects of several factors have not been considered so far, for example, how the presence of one or more competing person(s) along with the couple, who are in a love relation to each other, can influence the dynamics. Along the lines of the triangular love studies by Sprott [7, 8], it might be very interesting to investigate the role of S and D , in order to determine the steady-state relationship between a couple for the case of triangular love. Amongst many other interesting questions, one could also investigate how does a period of separation affect the system dynamics, within this modelling approach.

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